Session 1: Fibonacci and Spirals

Learning Outcomes

Upon completion of this session, students will:

- Be told that life obeys physical laws, just like any other process
- Know that nature has frequently occurring patterns, in flowers, animal stripes, plant shapes, etc.
- Be introduced to Fibonacci sequence through the rabbit growth example and know the rule for generating the Fibonacci sequence
- See how Fibonacci patterns are common in nature, exhibited through pictures of flowers, pinecones and pineapples
- See the occurrence of spirals in nature

Duration

90 minutes

Activities

The main activities in this session include a short lecture on the relevance of mathematics in biology and the existence of patterns in nature. This is followed by an activity on figuring out the Fibonacci sequence using a model of rabbit growth. After showing the students pictures of flowers whose petals follow the Fibonacci sequence, the last activity consists of demonstrating the presence of the Fibonacci sequence in pinecones and pineapples.

Materials

- Course handout, entitled "Mathematics and Biology" (see Appendix)
- Worksheets: "Busy Bunnies," "Pinecones and Spirals," and "Fibonacci Spirals" (see Appendix)
- PowerPoint presentation containing material discussed in this section **[included in the file Patterns_In_Nature_Session1.ppt]**

- Pinecones and pineapples. It is a good idea to get pinecones of different sizes, since the number of spirals in the pinecone depends on the size. Thus, different-sized pinecones can exhibit different pairs of Fibonacci numbers. Although we did not do this, flowers could also be presented as an exhibit.
- Permanent markers, to mark spirals onto the pineapples and pinecones

Background Information

Biology and Mathematics

The field of biology has changed a lot over the past 50 years. The discovery of DNA, the mapping of the human genome, and the development of other quantitative techniques have made mathematics an important part of biology. What does doing biology mean now? Not just looking at plants and animals, and characterizing and describing them, but trying to figure out how things work. In that sense, the motivation of a biologist is not very different from that of a person studying physics or chemistry; it is just that the questions they ask are mostly about living organisms instead of inanimate objects. But as we will see, there are more connections between the two than you think. What people thought before was that "life" and "living creatures" are completely out of the domain of analysis by physical laws. But as scientists are finding out more and more, they are realizing that this is not true. There are universal laws to be discovered in biology, too, and these might not be very different from what we get from studying the physical world.

Mathematics, on the other hand, does not just mean adding and subtracting. In a broader sense, mathematics is nothing but trying to figure out the universal laws in systems; 2 + 2 = 4 is just one of them. The new directions of mathematics take it into different fields: physics, chemistry and even biology. We want to look at biological systems and organisms and try to figure out why they are as they are. We want to find out how to characterize the properties of different organisms—e.g., growth, even emotions and maybe even "life." While we cannot do all these at this point, maybe some of you will! Scientists have progressed a lot in applying mathematics to the study of biology. We will see how amazing patterns in the real world can be identified, and we will try to think about why they occur. Often, we do not know the answers yet, but trying to think about an answer is fun, and that is what we will do in this course.

"Real mathematics is not crunching numbers but contemplating them—and the mystery of their connections." –Charles Krauthammer

Fibonacci Sequence

In this session, we will talk about the patterns that occur in nature. We will see how some patterns in flowers, plants and animals can be connected by a surprising set of numbers. We will also see how this leads us to apply mathematics to understand the growth of plants and some animals.

Many designs in nature are hard to understand but easy to recognize. You can draw a flower or tree, but can you say why it has to look that way? Biologists have a hard but exciting job trying to understand how nature works and why plants and other living creatures choose certain strategies. Usually, part of the answer is that nature is amazingly clever—life tends to find ways to get the most out of the resources available to it and usually takes the easiest path to its goals. Sometimes we are lucky to get a few clues about how this works from mathematics. One of those cases comes from something called the Fibonacci sequence. We will look at this special pattern of numbers and compare it to patterns found in nature.

A sequence is a chain of numbers generated by a particular rule. For example, 2, 4, 6, 8, 10… is a sequence made by adding 2 to the previous number. You can make a sequence using any rule you like (try it!), but some rules turn out to be more interesting than others. Fibonacci came up with a rule that has wonderful properties.

The idea is this: Start with 0 and 1. Make more numbers by adding the previous two together, like this:

$$0+1=1$$

$$1+1=2$$

$$1+2=3$$

$$2+3=5$$

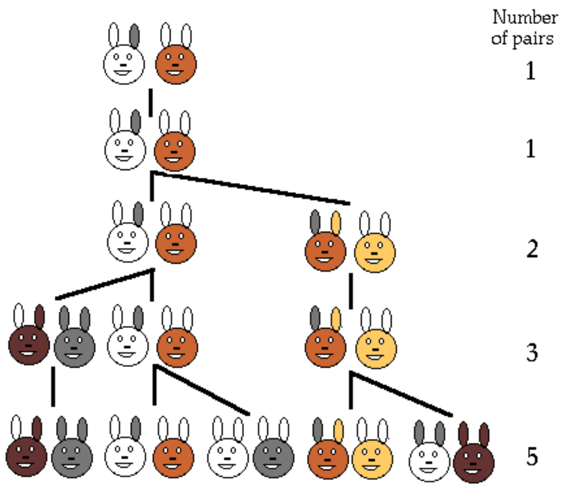
$$3+5=8$$

$$5+8=13$$

$$8+13=21$$

….

So the sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34….



Fibonacci was one of the greatest European mathematicians in medieval times. He lived around 1175 A.D. Fibonacci didn't start with the abstract idea just described, though. He originally wanted to figure out how a population of rabbits would grow (under very idealized circumstances). Let's try to see the sequence from that point of view.

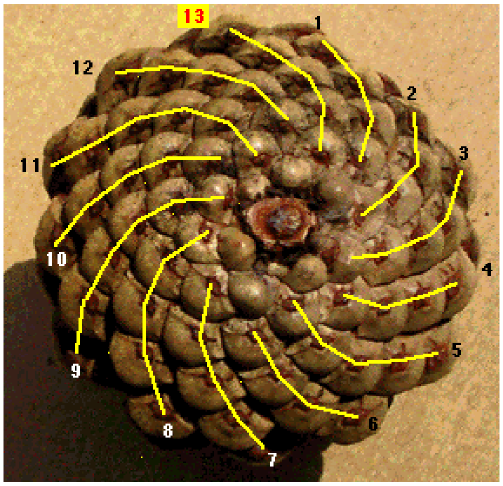
Pretend we get a young pair of rabbits and put them in a little park. The rabbits can't reproduce until they are two months old, so for the first month, we still just have one pair of rabbits. But starting the second month, the original pair reproduces, creating a second pair of rabbits. Each pair will make another pair every month it is able to do so. As can be seen from the picture, the number of pairs of rabbits in each generation follows what we now call the Fibonacci sequence.

There are other interesting ways to generate this sequence—the honeybee family tree is one. Check out www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html to find out how this works. (This website also contains a lot of the other information discussed in this section, including the rabbit diagram and the information below about the number of petals on different species of flowers.)

Fibonacci Sequence and Patterns

As we will see now, Fibonacci numbers are interesting to us for a completely different reason. Most flowers have petals that are Fibonacci numbers:

- 3 petals: lily, iris. Often, lilies have 6 petals formed from two sets of 3.
- 4 petals: very few plants show 4 petals (or sepals), but some, like fuchsia, do. 4 is *not* a Fibonacci number!
- 5 petals: buttercup, wild rose, larkspur, columbine (aquilegia), pinks. The humble buttercup has been bred into a multi-petaled form.
- 8 petals: delphiniums
- 13 petals: ragwort, corn marigold, cineraria, some daisies



- 21 petals: aster, black-eyed Susan, chicory
- 34 petals: plantain, pyrethrum
- 55, 89 petals: michaelmas daisies, the asteraceae family

Some species tend to have a very specific number of petals (e.g., buttercups), but others have a bit of variation from flower to flower, with the average being a Fibonacci number.

Fibonacci numbers are seen in a lot of seed heads and in pinecones. Take a pinecone, and view it as shown in the picture. There is a set of spirals going anticlockwise and a set of spirals going clockwise. Mark and count the number of spirals going each way. You will find that most often, the number of clockwise spirals and the number of anticlockwise spirals are two consecutive Fibonacci numbers. In sunflower seeds, too, you can find spirals going in both clockwise and anticlockwise directions. If you carefully count them, the numbers of these spirals are also Fibonacci.

Also, according to www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html (the website referenced above): "Many plants show the Fibonacci numbers in the arrangements of the leaves around their stems. If we look down on a plant, the leaves are often arranged so that leaves above do not hide leaves below. This means that each gets a good share of the sunlight and catches the most rain to channel down to the roots as it runs down the leaf to the stem… The Fibonacci numbers occur when counting both the number of times we go around the stem, going from leaf to leaf, as well as counting the leaves we meet until we encounter a leaf directly above the starting one. If we count in the other direction, we get a different number of turns for the same number of leaves. The number of turns in each direction and the number of leaves met are three consecutive Fibonacci numbers!"

Procedures

1. The first activity is about getting to know the Fibonacci numbers and the sequence.

2. After introducing the presence of patterns in nature, talk about the rabbit growth model. The rules are:
   a. We start with a pair of newborn rabbits, a male and a female.
   b. Each pair matures after two months and then starts producing babies every month.
   c. No rabbits die.
3. Following these rules, the teacher should work out the first couple of months. Using different colors to identify each unique pair is very helpful. After this, the students should be given a worksheet (see "Busy Bunnies" in the Appendix) that has the first 2/3 levels of the tree worked out. They should find out the number of rabbit pairs in the next few generations.
4. After checking that the students have worked out the next 2/3 generations correctly, the teacher writes down the sequence on the board and then invites suggestions about how the sequence could go on. Anyone giving a suggestion should be prompted to explain his or her rationale, too.
5. This is a good time to introduce Fibonacci the mathematician and his achievements.
6. Next, move straight into the activity of finding Fibonacci number in nature, including the "Pinecones and Spirals" worksheet in the Appendix. The teacher could use flowers here, in addition to pineapples and pinecones.
7. The pineapples and pinecones should have one spiral in each direction marked out. This makes it easier for the students to count the number of spirals in the clockwise and anticlockwise directions. Refer to the picture above to see how to mark out a spiral on a pinecone. The pair of numbers of spirals in the two directions is usually a pair of consecutive Fibonacci numbers. Often, some pinecones or pineapples might have ill-formed spirals, or the number might not be a Fibonacci number. It is important to explain this by mentioning that these are only some mathematical patterns that are largely true. Natural systems are extremely complex, and it is almost impossible to model them with 100% accuracy.

8. The last activity involves generating spirals using Fibonacci numbers. The student should be shown pictures of spiral galaxies, seashells, sunflower seed-heads, etc. Discussions could be invited at this point to try to figure out why the spiral shape is common. The teacher should point out that the spiral shape of the seashell is due to a different reason than the spiral shape of the galaxy. Yet, there is a common reason behind the spiral packing of the pinecones, pineapple, and sunflower seeds. Before going into that, we have to know something more about the properties of Fibonacci numbers. Use the "Fibonacci Spirals" worksheet in the Appendix. The students first need to figure out the missing lengths of the squares. Then, they should connect the big dots to create the spiral. The teacher should make a note that even though the squares are "growing," they fit in extremely well just because the lengths of the sides are Fibonacci numbers. (There will be more on this in the next session.)

Suggestions

The rabbit example is a bit simplistic. A more realistic and slightly more complicated example is the family of the honeybee, about which more information can be found at www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html. Fibonacci numbers also turn up in numerous other situations, and references should be easy to find on the Internet.